ANALYTICAL BUCKLING ANALYSIS FOR TOW-STEERED LAMINATES

UNIVERSITY OF DELAWARE | CENTER FOR COMPOSITE MATERIALS

Eli Bogetti, (MSDS)[2] , Prof. John W. Gillespie, Jr.[1]
University of Delaware | Center for Composite Materials[1] | Department of Data Science[2]

Introduction

- Composite materials are used extensively in the aerospace industry
- Provide weight reduction and strength and stiffness tailoring
- Tow-steering allows local stiffness tailoring and possibly more weight reduction
- Also introduces local mechanical coupling
- Buckling load is an important design consideration
- Rayleigh-Ritz method has been used to predicted

Rayleigh Ritz Method

- Assume displacement functions

$$u(x,y), \qquad v(x,y), \qquad w(x,y)$$

- Minimize total potential energy (Π)

$$\Pi = U + V$$

  - Plate's strain energy: U
  - Energy from external forces: V

$$\frac{\partial \Pi}{\partial S_{mn}} = 0$$

  - S_{mn} are unknowns in u, v, w
- Solve eigenproblem

$$([L] - N_{crit}[R])\{S\} = \{0\}$$

  - Eigenvalue – Critical buckling load
  - Eigenvector – Mode shape

Isotropic Case

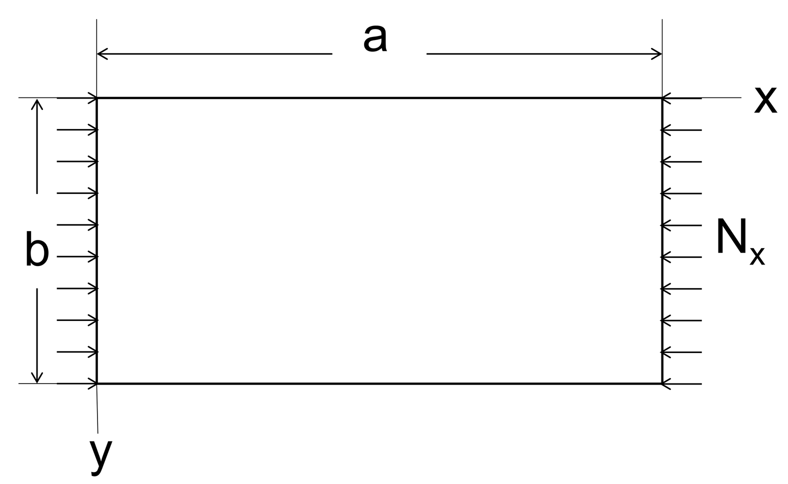
- For unsymmetric cross-ply laminates:

$$u = \sum_m \sum_n U_{mn} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$v = \sum_m \sum_n V_{mn} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right)$$

$$w = \sum_m \sum_n W_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

- Timoshenko and Gere[1] simply supported steel plate under uniform axial compression

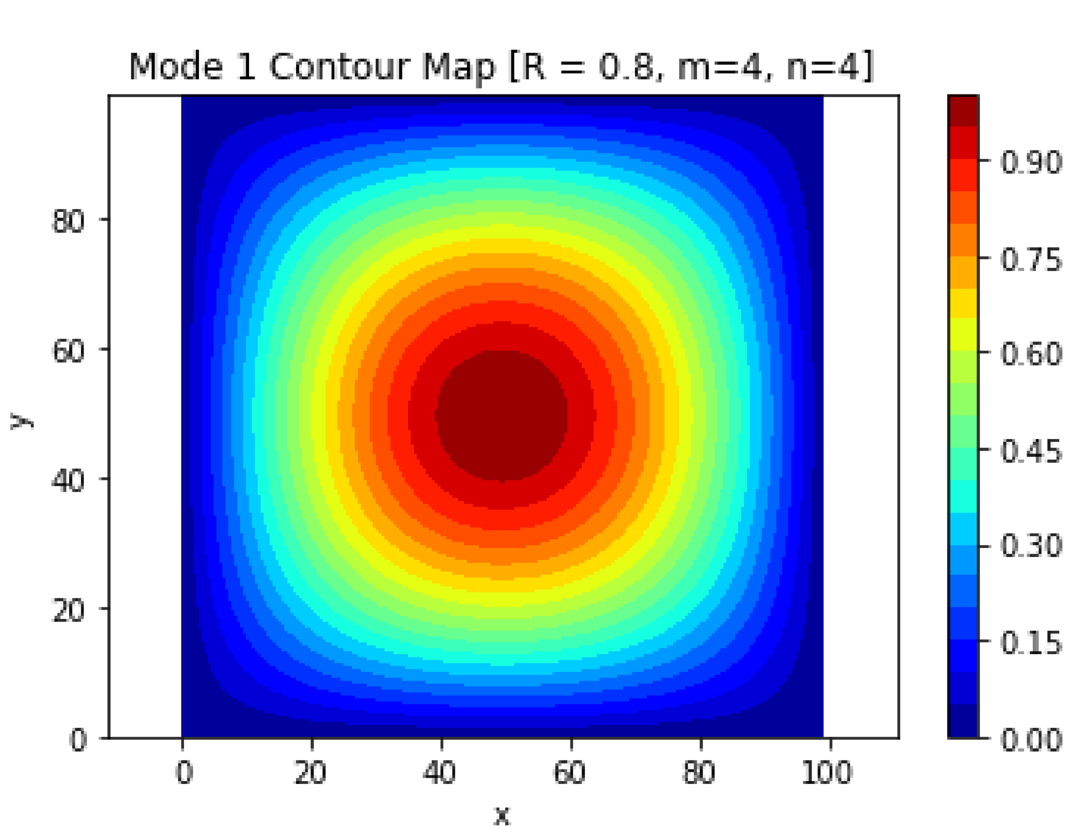


- $E = 206.8\ Gpa$
- $\nu = 0.3$
- $^{b}/_{h} = 100$

- Results compared with nondimensional buckling coefficient k

Nondimensional buckling coefficient k for various aspect ratios

a/b	0.4	0.6	0.8	1.0
Timoshenko & Gere	8.41	5.14	4.20	4.00
Predicted	8.41	5.14	4.20	4.00



Mechanical Coupling

- Complicates buckling analysis and requires more accurate assumed displacement functions
- For antisymmetric angle-ply

$$u = \sum_m \sum_n U_{mn} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right)$$

$$v = \sum_m \sum_n V_{mn} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$$

$$w = w_{cross-ply}$$

- Assumed displacement function

$$u = U_1 \frac{x}{a} + u_{cross-ply} + u_{angle-ply}$$

$$v = V_1 \frac{x}{a} + V_2 \frac{y}{b} + v_{cross-ply} + v_{angle-ply}$$

$$w = w_{cross-ply} = w_{angle-ply}$$

- Compared assumed displacement functions on different laminates[2]
  - Hercules AS4/3501-6 graphite/epoxy
  - Isolated certain coupling terms

Laminates and associated coupling terms

Laminate	Coupling Terms
Timoshenko	None
$[0_3/90_3/0_3/90_3]$	B_{11}, B_{12}, B_{22}
$[0_2/45_2/0_2/45_2/0_2]$	$A_{16}, A_{26}, D_{16}, D_{26}$
$[0_2/45_2/0_2/-45_2/0_2]$	B_{16}, B_{26}
$[0_6/45_6]$	Full ABD

Nondimensional buckling coefficient k

Laminate	Bogetti	Cross-Ply	Angle-Ply
Timoshenko	4.00	4.00	4.00
$[0_3/90_3/0_3/90_3]$	1.02	1.02	1.16
$[0_2/45_2/0_2/45_2/0_2]$	1.30	1.30	1.30
$[0_2/45_2/0_2/-45_2/0_2]$	1.13	1.28	1.13
$[0_6/45_6]$	0.78	0.92	1.06

Future Work

- Finalize validation for mechanical coupling cases
- Rework problem to solve for u and v to compare to assumed displacement functions
- Incorporate tow-steering

$$\theta(x) = \begin{cases} 2(T_0 - T_1)\left(\frac{x}{a}\right) + T_1 & 0 \le x \le \frac{a}{2} \\ 2(T_1 - T_0)\left(\frac{x - \frac{a}{2}}{a}\right) + T_0 & \frac{a}{2} < x \le a \end{cases}$$

  - Fiber angle at panel center: T_0
  - Fiber angle at panel ends: T_1
- Investigate Rayleigh-Ritz effect on capturing local mechanical coupling

References

- Timoshenko and Gere, *Theory of Elastic Stability.* 1961[1]
- David Jensen, *Buckling and Postbuckling Behavior of Unbalanced and Unsymmetric Laminated Graphite/Epoxy Plates*. 1986[2]
- Papazoglou, *Buckling of unsymmetric laminates under linearly varying, biaxial in-plane loads, combined with shear.* 1992

Acknowledgements

This material is based upon work supported by the National Aeronautics and Space Administration under Grant and Cooperative Agreement No. 80NSSC20M0164, issued through the Aeronautics Research Mission Directorate, Transformative Aeronautics Concepts Program, University Leadership Initiative.

Special thanks to my mentor, Brian Mason, and the ISAAC team at NASA Langley.