ON THE SCALING OF STRUCTURAL RESPONSE ON THE BALLISTIC IMPACT

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Objectives and goals

- Due to complexity and high cost of the realistic armor systems, there is constant interest in using smaller and simplified scaled models to get information about the behavior of original systems. Scaled modeling is often used to evaluate the performance of a particular design, without incurring the expense of a full-sized prototype. Similarly, the use of scaled down models can drastically decrease the cost of investigating the response of engineering structures under ballistic impact. However, the application of scaled models for predicting the response of a complex structure is limited due to the absence of a consecutive method to "translate" the data measured on the scaled-down model to real-size structures. Thus, the systematic investigation of ballistic event and structural response scaling is of interest.

Background of scaling procedure

A. What is scaling?

Pi-theorem lies in the background of a physically reasonable scaling methods. The theorem states that if we have a physically meaningful equation involving a certain number, n, of physical variables, and these variables are expressible in terms of k independent fundamental physical quantities, then the original expression is equivalent to an equation involving a set of $p = n - k$ dimensionless variables constructed from the original variables. From physical point of view, the systems, charactering with the same set of dimensionless parameters, are similar in the same sense as in elementary geometry are similar triangles having the same angles.

Let suggest that we consider linear distributed isotropic body, having some shape which can be characterized by geometrical sizes $L_1, L_2, \ldots L_N$

$$\mathbf{F}(t,\mathbf{X}') \rightarrow \boxed{\langle \alpha L_1, \alpha L_2, \ldots \alpha L_N \rangle \quad c_l = \sqrt{\frac{E(1-\sigma)}{\rho(1+\sigma)(1-2\sigma)}} \quad c_t = \sqrt{\frac{E}{2\rho(1+\sigma)}}} \rightarrow \mathbf{U}(t,\mathbf{X}'')$$

How Pi-theorem is working in application to wave propagation in complex structures?

Scaling of the elastic waves-1

A. Scaling of equations:

Let us consider propagations of linear elastic waves in isotropic homogeneous material:

$$\ddot{\mathbf{u}} - c_t^2 \Delta \mathbf{u} - \left(c_l^2 - c_t^2\right) \nabla div\mathbf{u} = \frac{1}{\rho} \mathbf{f}(\mathbf{x},t) \quad c_l = \sqrt{\frac{E(1-\sigma)}{\rho(1+\sigma)(1-2\sigma)}} \quad c_t = \sqrt{\frac{E}{2\rho(1+\sigma)}} \quad \left(\frac{c_l}{c_t}\right)^2 = \frac{2(1-\sigma)}{(1-2\sigma)}$$

In space-time form scaling law is not obvious. To make it clear, it is convenient to make Fourier transform and consider $(\omega, \mathbf{x})$ domain:

$$k_0^2 \mathbf{u} + \frac{c_t^2}{c_l^2} \Delta \mathbf{u} + \left(1 - \frac{c_t^2}{c_l^2}\right) \nabla div\mathbf{u} = -\frac{1}{\rho c_l^2} \mathbf{f}_\omega(\mathbf{x}) \xrightarrow{k_0^2 = \frac{\omega^2}{c_l^2}} \boxed{\mathbf{u}_\omega(\xi,\omega) + \frac{c_t^2}{c_l^2} \Delta_\xi \mathbf{u}_\omega(\xi,\omega) + \left(1 - \frac{c_t^2}{c_l^2}\right) \nabla_\xi div_\xi \mathbf{u}_{\omega(\xi,\omega)} = -\frac{1}{\rho\omega^2} \mathbf{f}_\omega(\xi) = -\mathbf{F}_M(\xi,\omega)}$$

B. Similarity parameters: $\xi = k_0 \mathbf{x} = \frac{\omega}{c_l} \mathbf{x} \quad \eta = \frac{\mathbf{f}_\omega(k_0 \mathbf{x})}{\rho \omega^2}$

Similar systems are characterizing by the SAME GEOMETRY IN DIMENSIONLESS COORDINATES. Thus, if the model is made from the same material as the original object and all dimensions of the scaled model are α times of initial coordinates, one must proportionally change the characteristic times of all processes *and proportionally change the acting forces.*

$$\Delta_\xi \nabla \cdot \mathbf{u}_\omega(\xi,\omega) + \nabla \cdot \mathbf{u}_\omega(\xi,\omega) = \boxed{-\frac{1}{\rho\omega^2} div_\xi \mathbf{f}_\omega(\xi)} \qquad \nabla \times \mathbf{u}_\omega(\xi,\omega) + \boxed{\frac{2(1-\sigma)}{(1-2\sigma)}} \Delta_\xi \nabla \times \mathbf{u}_\omega(\xi,\omega) = -\frac{1}{\rho\omega^2} \nabla \times \mathbf{f}_\omega(\xi)$$

Boundary conditions also must be presented in dimensionless form

Scaling of elastic waves-II

B. Dimensionless Green's function. Green's function takes in account specific boundary conditions.

$$\Delta_\xi G_{l0}(\xi',\xi) + G_{l0}(\xi',\xi) = \delta(\xi) \qquad \text{Longitudinal waves}$$

$$\Delta_\xi \mathbf{G}_{s0}(\xi'-\xi) + \frac{2(1-\sigma)}{(1-2\sigma)} \mathbf{G}_{s0}(\xi'-\xi) = \delta(\xi) \qquad \text{Shear waves}$$

If Green's functions are found, one can present, for example, longitudinal field in the "universal" coordinates as:

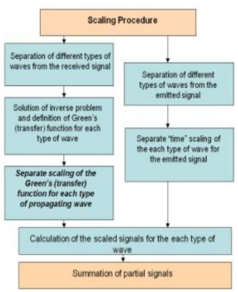
$$\left(\nabla_\xi \cdot \mathbf{u}_\omega\right) = -\frac{1}{\rho\omega^2} \int G_l(\xi,\xi') div\mathbf{f}_\omega(\xi') d\Omega'$$

Thus, the (longitudinal) field in actual coordinates can be presented as:

$$\nabla \cdot \mathbf{u} = -\frac{1}{\sqrt{2\pi}} \int e^{i\omega t} \int G_l\left(\omega, \frac{\omega}{c_l}\mathbf{x}, \frac{\omega}{c_l}\mathbf{x}'\right) div\mathbf{f}_\omega\left(\frac{\omega}{c_l}\mathbf{x}'\right) d\Omega' d\omega$$

This obvious consideration identifies the major goal of experiments with proportionally sub-scaled models: to identify dimensionless Green's function of the system. In this case scaling up looks relatively trivial. *Also, it is necessary to point out that the last expression is more general, than initial equations.*

Scaling Procedure



In the frame of standard Bode representation, the transfer function *for each trajectory* must be presented in the form, which one can consider as representation of the Green 's function for taken point :

$$K(\omega) = k(r_1,r_2) \frac{(\omega - \omega_{z0}(r_1,r_2))(\omega - \omega_{z1}(r_1,r_2)) \ldots (\omega - \omega_{zN}(r_1,r_2))}{(\omega - \omega_{p0}(r_1,r_2))(\omega - \omega_{p1}(r_1,r_2)) \ldots (\omega - \omega_{pM}(r_1,r_2))}$$

For each identified frequency, the model scaling coefficient must be identified and content of taken harmonic in the affecting force must be proportionally scaled up.

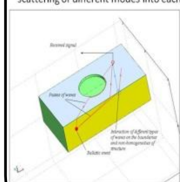
However, effect of complexity of realistic systems is not yet well understood.

If the density of significant modes becomes so high, that number of modes becomes big, one must consider the system in stochastic terms, as response of the system of a big number of oscillators. Such approach to scaling we will consider somewhere else.

Conclusions

Major conclusion from physical consideration are:

- The physically consistent physical modeling requires exact scaling of the object of interest in all three spatial dimensions, including fine elements of structure which can affect propagation of waves in the frequency range of interest.
- In the mean time, scaling of ballistic shock itself looks practically very difficult because there are limited ability to scale simultaneously generated by shock acoustic signal and other parameters as .
- Separate scaling of the "shape" and wall thickness leads to the problem of separate scaling of propagation of different types of waves which is formally impossible if one cannot neglect the scattering of different modes into each other.



What can we do?

1. For low-frequency range related with overall structural response spectral analyses is necessary .
2. For high frequency modes, experimental set-up must be able to identify direction from where the signal came and types of waves.
3. Special scaling procedure, which includes measurement of frequency specter nearby the source and in the points of interest with "re-summation" of the harmonics must be applied

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