On the Continuum Theory of Conformable Robots

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INTRODUCTION

What They Are: Conformable Robots?

The DARPA request states that ChemBots should be,

Soft, flexible, mobile objects that can identify and maneuver through openings smaller than their static structural dimensions; reconstitute size, shape, and functionality after traversal; carry meaningful payloads; and perform tasks.

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"nature provides many examples of ChemBot functionality. Many soft creatures, including mice, octopi, and insects, readily traverse openings barely larger than their largest 'hard' component."

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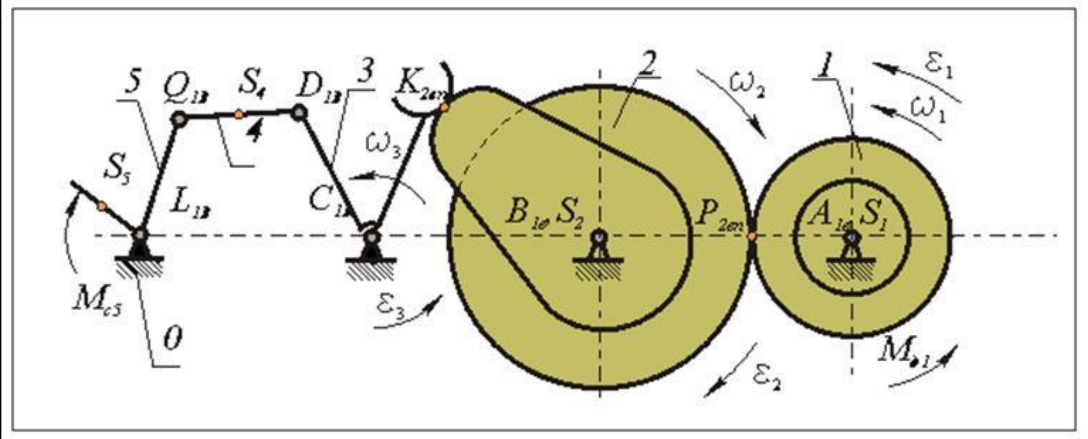
OBJECTIVES AND GOALS

Conformable robots offer the potential to provide vast improvements in degrees of flexibility to the point where it would need to act almost as a droplet of liquid. Macro-analogue include octopus tentacles, which demonstrates extremely high level of flexibility, particularly because they have jelly-like composition. In contrast to "discrete robots", these devices will have distributed "cell" structures. Each cell can potentially include an "engine" (or an active component based on a class of smart materials) and actively change it's shape under a control signal. Furthermore, the set of cells will form a quasi-continuum (in general case, anisotropic) structure where each cell interacts with it neighbors.

The goal of the work is to formulate enough general approach which permit by regular way to describe the behavior of active continuum structures.

APPROACH TO THE THEORY-I

Theory of conformable robot generalizes the theory of continuums with internal structure.



Regular theory of machines:

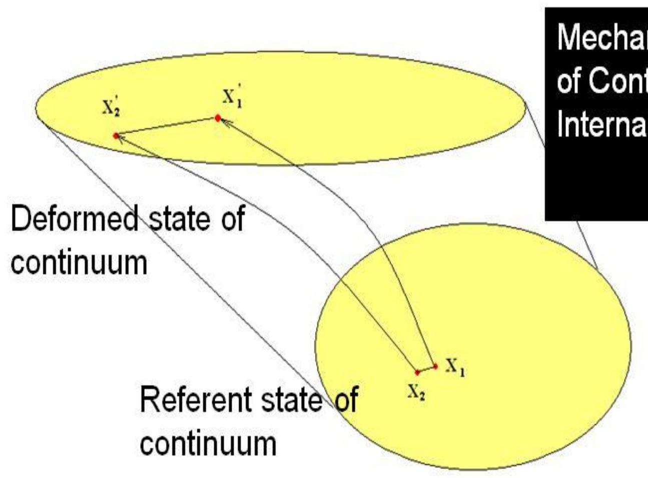
$$\sum_i \mathbf{F}_i + \sum_i \mathbf{F}_{i(inert)} = 0$$

$$\sum_i \mathbf{M}_i + \sum_i \mathbf{M}_{i(inert)} = 0$$

How to approach a theory of continuum objects, each point of which is a machine? Let recall that generalized force is a derivative of the work (energy) over generalized coordinate of the system. For example, regular force is a derivative of the work (energy) over spatial coordinate, angular momentum is a derivative of the work over the angle. Generalization of this idea on the dynamics is the Lagrangian approach in frame of which the Action of the system must be defined in terms of generalized coordinates.

APPROACH TO THE THEORY-II

Mechanical Background: Representation of Continuum robots as a Continuums with Internal structure.



Passive analogies:

Cosserat Elasticity

$\sigma_{kl} = \lambda\varepsilon_{rr}\delta_{kl} + (2\mu + \kappa)\varepsilon_{kl} + \kappa\varepsilon_{klm}(r_m - \phi_m)$

$m_{kl} = \alpha\,\phi_{r,r}\,\delta_{kl} + \beta\,\phi_{k,l} + \gamma\,\phi_{l,k}$

in which σ_{kl} is the force stress (which is a symmetric tensor in classical elasticity but is asymmetric here), m_{kl} is the couple stress (or moment per unit area), $\varepsilon_{kl} = (u_{k,l} + u_{l,k})/2$ is the small strain, u is the displacement, and e_{klm} is the permutation symbol.

The microrotation ϕ_k in Cosserat elasticity is kinematically distinct from the macrorotation $r_k = (e_{klm}u_{m,l})/2$.

External parameters for cell-machine are components of ***macro-strain tensor*** and (dynamically) ***tensor of macro-rotation***.

Internal parameters are parameters of micro-machine (parameters of micro-robot)

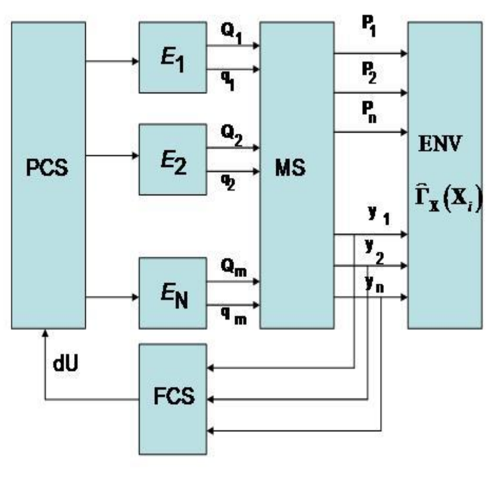
$$\xi_s = \xi_s(\mathbf{X},t)$$

$$\hat{\Gamma}_{\mathbf{X}}(\mathbf{X}_i) = \left\| \left(\frac{\partial \xi_{Si}}{\partial X_j} \right)_{\mathbf{X}_i} \right\|$$

FUNCTIONAL DIAGRAM OF CONFORMABLE ROBOT

Relationship between macro-continuum behavior and behavior of taken macro-point (micro-robot) are establishing by *axiom* of locality:

There exist a map, relating the set of external parameters with the space of internal parameters of taken micro-robot. It is generalization of Cauchy-Born hypotheses from continuum theory of crystals.



We denoted: PCS – is the Program Control System; FCS – is the Feedback Control System; MS – is the mechanical system of machine-cell, ENV – is the environment; E_i – are controlled energy sources, $\mathbf{Q}_i$ – are generalized forces related with generalized coordinates $\mathbf{q}_i$-of the mechanical system; $\mathbf{P}_i$- is generalized forces conjugated to the external coordinates y_i .

Program control system provides the signal to the energy sources, while active generalized forces **Q** are associated with the mechanical degrees of freedom $\mathbf{q}_i$ of the mechanical system. Mechanical system transfers these signals into the generalized forces related with macro-deformation of the cell described by the external coordinates.

Functional diagram of the local multi-engine machine-cell

GOVERNING EQUATIONS

$$\frac{d\rho}{dt} + \rho\frac{\partial \dot{X}_i}{\partial x_i} = 0$$ Mass conservation

$$\frac{d}{dt}\left(\rho\frac{\partial l}{\partial \dot{X}_k}\right) - \frac{\partial \Sigma_{kj}}{\partial x_j} = Q_k$$ External momentum conservation

$$\frac{d\rho\Psi_r}{dt} = \rho f_r$$ Newtonian equation for internal degrees of freedom

$$\frac{dy_k}{dt} = C_{rk}(y_k, p_l, e_r, \mathbf{X})\frac{de_r}{dt} + D_{rk}(y_k, p_l, e_r, \mathbf{X})\frac{dp_r}{dt}$$ Structural equations

$$E_r\left(\left(\mathbf{X}(x,t) - \bar{\mathbf{X}}(x,t)\right), t\right)$$ Control rule

$$\Sigma_{ij} = -\left(\rho F_{ij} - \frac{d\rho P_{ij}}{dt}\right)$$ Definition of stress

$$Q_k = \rho\frac{\partial l}{\partial X_k}$$ Definition of external force (like gravity)

CONCLUSIONS

- Lagrangian approach based on the ideas of continuum with internal degrees of freedom permits consecutively develop continuum approach to the modeling of conformable robots with arbitrary mechanical properties and control algorithms.
- Obtained general equations of mechanics of conformable robots show significant similarity with the mechanics of general (anisotropic) materials and materials with internal degrees of freedom
- This equations can be used for solution of any problems of the dynamics of the conformable robots and their interaction with environment.
- On the the base of continual approach appropriate algorithms of robotic control can be developed and these algorithms will be different the algorithms of discrete robots, because they will be based on the continuum equations of the robotodynamics.