ANALYTICAL BUCKLING ANALYSIS FOR TOW-STEERED LAMINATES

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Introduction
• Composite materials are used extensively in the aerospace industry
• Provide weight reduction and strength and stiffness tailoring
• Tow-steering allows local stiffness tailoring and possibly more weight reduction
• Also introduces local mechanical coupling
• Buckling load is an important design consideration
• Rayleigh-Ritz method has been used to predicted

Rayleigh Ritz Method
• Assume displacement functions
  \[ u(x,y), \quad v(x,y), \quad w(x,y) \]
• Minimize total potential energy (\(\Pi\))
  \[ \Pi = U + V \]
  - Plate’s strain energy: \(U\)
  - Energy from external forces: \(V\)
  \[ \frac{\partial \Pi}{\partial S_{mn}} = 0 \]
  - \(S_{mn}\) are unknowns in \(u,v,w\)
• Solve eigenproblem
  \[ \{L\} - N_{\text{crit}}\{R\}\{S\} = \{0\} \]
  - Eigenvalue – Critical buckling load
  - Eigenvector – Mode shape

Isotropic Case
• For unsymmetric cross-ply laminates:
  \[
  u = \sum_{m}^{\infty} \sum_{n}^{\infty} U_{mn} \cos \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right) \\
  v = \sum_{m}^{\infty} \sum_{n}^{\infty} V_{mn} \sin \left( \frac{m\pi x}{a} \right) \cos \left( \frac{n\pi y}{b} \right) \\
  w = \sum_{m}^{\infty} \sum_{n}^{\infty} W_{mn} \sin \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right)
  \]
• Timoshenko and Gere¹ simply supported steel plate under uniform axial compression

Nondimensional buckling coefficient \(k\) for various aspect ratios

<table>
<thead>
<tr>
<th>(a/b)</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
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<tbody>
<tr>
<td>Timoshenko</td>
<td>8.41</td>
<td>5.14</td>
<td>4.20</td>
<td>4.00</td>
</tr>
<tr>
<td>Predicted</td>
<td>8.41</td>
<td>5.14</td>
<td>4.20</td>
<td>4.00</td>
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</tbody>
</table>

Mechanical Coupling
• Complicates buckling analysis and requires more accurate assumed displacement functions
• For antisymmetric angle-ply
  \[
  u = \sum_{m}^{\infty} \sum_{n}^{\infty} U_{mn} \sin \left( \frac{m\pi x}{a} \right) \cos \left( \frac{n\pi y}{b} \right) \\
  v = \sum_{m}^{\infty} \sum_{n}^{\infty} V_{mn} \cos \left( \frac{m\pi x}{a} \right) \sin \left( \frac{n\pi y}{b} \right) \\
  w = w_{\text{cross-ply}}
  \]
• Assumed displacement function
  \[
  u = U_1 \frac{x}{a} + u_{\text{cross-ply}} + u_{\text{angle-ply}} \\
  v = V_1 \frac{y}{b} + v_{\text{cross-ply}} + v_{\text{angle-ply}} \\
  w = w_{\text{cross-ply}} = w_{\text{angle-ply}}
  \]
• Compared assumed displacement functions on different laminates²
  - Hercules AS4/3501-6 graphite/epoxy
  - Timoshenko and Gere, Theory of Elastic Stability. 1961
• Theory of Elastic Stability.

Nondimensional buckling coefficient \(k\)

<table>
<thead>
<tr>
<th>Laminate</th>
<th>Cross-Ply</th>
<th>Angle-Ply</th>
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<tbody>
<tr>
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<td>4.00</td>
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<td>[0₂/45/0₂]</td>
<td>0.78</td>
<td>0.92</td>
</tr>
</tbody>
</table>

References
• Timoshenko and Gere, Theory of Elastic Stability. 1961¹
• David Jensen, Buckling and Postbuckling Behavior of Unbalanced and Unsymmetric Laminated Graphite/Epoxy Plates. 1986²
• Papazoglou, Buckling of unsymmetric laminates under linearly varying, biaxial in-plane loads, combined with shear. 1992

Future Work
• Finalize validation for mechanical coupling cases
• Rework problem to solve for \(u\) and \(v\) to compare to assumed displacement functions
• Incorporate tow-steering
• Investigate Rayleigh-Ritz effect on capturing local mechanical coupling

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