ON THE EFFECTIVE ELASTIC PROPERTIES OF MICROCRACKED MATERIALS

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INTRODUCTION

- Microcracks are common defects in solid materials. In addition multiple cracks usually coexist in a single solid.
- The prediction of effective elastic properties of a microcracked solid is technically challenging and can find many practical applications.
- Up to now many different methods have been developed to predict the effective stiffness of a cracked solid.
- We will first summarize these prediction methods and then the problem to be studied is raised and our approach and result are shown.

ANALYTICAL MODELS

- **Self-consistent method (SCM):** A crack is directly embedded into an effective medium (Budiansky and O'Connell, 1976);
- **Generalized self-consistent method (GSCM):** A crack is surrounded by a matrix shell, and then embedded in the effective medium (Aboudi and Benveniste, 1987; Santare et al., 1995);
- **Mori-Tanaka method (MTM):** (Benveniste, 1986);
- **Differential scheme method (DS):** (Salganik, 1973; Zimmermann, 1985; Hashin, 1988);
- **Modified differential scheme (MDS):** (Sayers and Kachanov, 1991).

COMPUTATIONALLY INTENSIVE APPROACHES

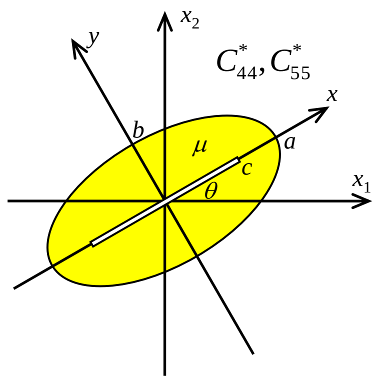
- **GSCM plus finite element method (FEM):** This numerical method can easily take into account crack face contact and friction. Only plane stress problems were addressed (Su et al., 2007);
- **Representative unit cell approach:** The microgeometry of a cracked solid is modeled by a periodic structure with unit cell containing multiple cracks. The method combines the principle of superposition, technique of complex potentials and results in the theory of special functions. Only open cracks were treated. Anti-plane shear and plane strain problems were considered (Kushch et al., 2009).

EFFECTIVE MODULI OF MICROCRACKED MATERIALS UNDER ANTIPLANE LOADING

The strain energy balance relationship under anti-plane shear deformation:

$$\frac{\left(\sigma_{31}^{0}\right)^2}{2C_{55}^{*}}+\frac{\left(\sigma_{32}^{0}\right)^2}{2C_{44}^{*}} =\frac{\left(\sigma_{31}^{0}\right)^2+\left(\sigma_{32}^{0}\right)^2}{2\mu}+\frac{\eta}{2c^2}\int_{-\theta_0}^{\theta_0}\phi(\theta)\int_{C_k}[u_3]t_3^0\,dC_k\,d\theta,$$

THE GSCM



THE RESULT

$$\frac{\mu}{C_{55}^{*}}=1+\frac{a\mu^{*}+bC_{44}^{*}}{(a+b)\mu^{*}}\frac{\pi\eta[2\theta_0-\sin(2\theta_0)]}{2\theta_0\left[1+\Gamma-R^{-2}(1-\Gamma)\right]},$$

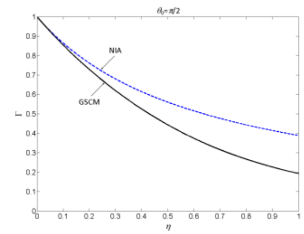
$$\frac{\mu}{C_{44}^{*}}=1+\frac{a\mu^{*}+bC_{55}^{*}}{(a+b)\mu^{*}}\frac{\pi\eta[2\theta_0+\sin(2\theta_0)]}{2\theta_0\left[1+\Gamma-R^{-2}(1-\Gamma)\right]},$$

where

$$\eta=\frac{c^2}{\pi ab}=\frac{4}{\pi(R^2-R^{-2})}$$

Remark: An approximation of GSCM is pursued when the undamaged solid is anisotropic.

THE RESULT



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