EFFICIENT ANALYSIS OF SCATTERING FROM PERIODIC STRUCTURES COMPOSED OF ARBITRARY INHOMOGENOUS AND ANISOTROPIC MATERIALS USING FE-BI METHOD ACCELERATED BY FFT

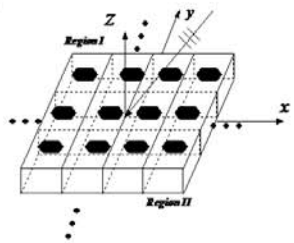
S. Cui (PD) and D. S. Weile

University of Delaware • Center for Composite Materials • Department of Electrical and Computer Engineering

Introduction

- Periodic structures are useful in a variety of applications
  - Absorbers
  - Frequency selective surfaces
  - Photonic band gap structures
  - Filters
- Periodic structures may involve inhomogeneous & anisotropic materials
- To analyze periodic structures, a scheme is required that
  - Easily enforces periodic boundary conditions
  - Handles complex inhomogeneous and anisotropic materials
  - Is efficient with respect to both memory and time
- The objective of this study: Efficient computation of the reflection and transmission coefficients of periodic structures

Introduction (cont.)



$$\bar{\varepsilon}_r(\vec{r}) = \begin{bmatrix} \varepsilon_{xx}(\vec{r}) & \varepsilon_{xy}(\vec{r}) & \varepsilon_{xz}(\vec{r}) \\ \varepsilon_{yx}(\vec{r}) & \varepsilon_{yy}(\vec{r}) & \varepsilon_{yz}(\vec{r}) \\ \varepsilon_{zx}(\vec{r}) & \varepsilon_{zy}(\vec{r}) & \varepsilon_{zz}(\vec{r}) \end{bmatrix}$$

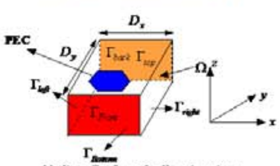
$$\bar{\mu}_r(\vec{r}) = \begin{bmatrix} \mu_{xx}(\vec{r}) & \mu_{xy}(\vec{r}) & \mu_{xz}(\vec{r}) \\ \mu_{yx}(\vec{r}) & \mu_{yy}(\vec{r}) & \mu_{yz}(\vec{r}) \\ \mu_{zx}(\vec{r}) & \mu_{zy}(\vec{r}) & \mu_{zz}(\vec{r}) \end{bmatrix}$$

- The Finite Element–Boundary Integral (FE-BI) Method is applied to analyze scattering from the periodic structure. Advantages of FE-BI include:
  - Easy enforcement of periodic boundary conditions,
  - Easy application to inhomogeneous and arbitrary geometries,
  - Sparsity of matrices for FE part
  - Simple formulation

Introduction (cont.)

- Present difficulties
  - Memory consumption of BI submatrix (dense)
  - Slow computations arising from BI submatrix:
    - Dense matrix-vector multiplication
    - Green's function computation
- Approaches to overcome the difficulties above
  - Uniform Mesh for BI surfaces
    - Block-Toeplitz storage technique
    - FFT-based matrix-vector multiplication
  - The Ewald transformation
- Outline
  - Introduction
  - Finite Element-Boundary Integral (FE-BI) Method for Periodic Structures
  - Numerical results
  - Conclusions

FE-BI Method (1)



Unit cell of periodic structure

$$\vec{E}(\Gamma_{right}) = \vec{E}(\Gamma_{left})\exp(jk_{x,0}D_x)$$
$$\vec{E}(\Gamma_{back}) = \vec{E}(\Gamma_{front})\exp(jk_{y,0}D_y)$$

Dirichlet boundary condition on PEC

$$F(\vec{W},\vec{E}) = \int_\Omega \left[\nabla\times\vec{W}^*\cdot(\bar{\mu}_r^{-1}\nabla\times\vec{E}) - k_0^2\bar{\varepsilon}_r\vec{W}^*\cdot\vec{E}\right]dV - jk_0\eta_0\int_{\Gamma_{top}+\Gamma_{bottom}}\vec{W}^*\cdot(\hat{n}\times\vec{H})dS = 0$$

FE-BI Method (2)

- The boundary integral equation is given by

$$\frac{1}{2}\vec{H} + \frac{jk_0}{\eta_0}\int\left[\vec{E}(\vec{r}')\times\hat{n}G_p(\vec{r},\vec{r}') + \frac{1}{k_0^2}\nabla'(\vec{E}(\vec{r}')\times\hat{n})\nabla G_p(\vec{r},\vec{r}')\right]ds = \begin{cases}\vec{H}^{inc} & \text{on } \Gamma_{top}\\ 0 & \text{on } \Gamma_{bottom}\end{cases}$$

where the periodic Green's function is

$$G_p(\vec{r},\vec{r}') = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{\exp[-jk_{xm}(x-x') - jk_{yn}(y-y')]}{2jD_xD_yk_{zmn}}$$

- This is a slowly convergent infinite series, to alleviate computational expense of the infinite sum, the Ewald transformation is used:

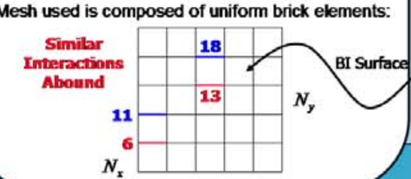
$$G_p(\vec{r},\vec{r}') = G_{p1}(\vec{r},\vec{r}') + G_{p2}(\vec{r},\vec{r}')$$
$$G_{p1}(\vec{r},\vec{r}') = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{\exp\left[-jk_{xm}(x-x') - jk_{yn}(y-y')\right]}{2jD_xD_yk_{zmn}}\operatorname{erfc}(jk_{zmn}/2E)$$
$$G_{p2}(\vec{r},\vec{r}') = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{\exp\left[jk_{x,0}mD_x + jk_{y,0}nD_y\right]}{8\pi R_{mn}}\times \left[\exp(jk_0R_{mn})\operatorname{erfc}(R_{mn}E + jk_0/2E) + \exp(-jk_0R_{mn})\operatorname{erfc}(R_{mn}E - jk_0/2E)\right]$$

FE-BI Method (3)

- By using the Ewald transform, the infinite series converges extremely rapidly **M=N=2**
- The variational functional for the FE is given by

$$F(\vec{W},\vec{E}) = \int_\Omega\left[\nabla\times\vec{W}^*\cdot(\bar{\mu}_r^{-1}\nabla\times\vec{E}) - k_0^2\bar{\varepsilon}_r\vec{W}^*\cdot\vec{E}\right]dV + 2jk_0\eta_0\int_{\Gamma_{top}}\hat{n}\times\vec{W}^*(\vec{r})\cdot\vec{H}^{inc}(\vec{r})ds$$
$$-2k_0^2\int_{\Gamma_{top}+\Gamma_{bottom}}\int_{\Gamma_{top}+\Gamma_{bottom}}G_p(\vec{r},\vec{r}')\begin{bmatrix}\hat{n}\times\vec{W}^*(\vec{r})\cdot\hat{n}\times\vec{E}(\vec{r}') - \\ \nabla\cdot\hat{n}\times\vec{W}^*(\vec{r})\nabla\cdot\hat{n}\times\vec{E}(\vec{r}')/k_0^2\end{bmatrix}ds\,ds'$$

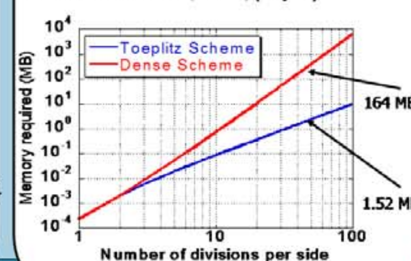
- Mesh used is composed of uniform brick elements:



Memory Savings

Dense scheme $\left[N_x(N_y+1)+N_y(N_x+1)\right]^2$

Toeplitz scheme $16(2N_x-1)(2N_y-1)$



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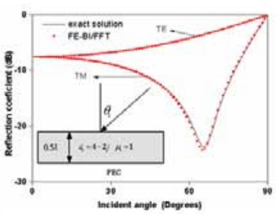
Numerical Results (1)

- All of the numerical results presented here refer to a nonmagnetic material.
- Also, all of the results presented here assume that the axes of anisotropy are aligned with the axes of periodicity.

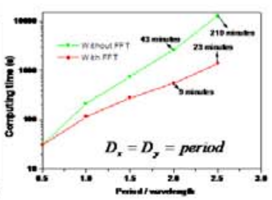
$$\bar{\varepsilon}_r(\bar{r}) = \begin{bmatrix} \varepsilon_{xx}(\bar{r}) & 0 & 0 \\ 0 & \varepsilon_{yy}(\bar{r}) & 0 \\ 0 & 0 & \varepsilon_{zz}(\bar{r}) \end{bmatrix} \quad \bar{\mu}_r(\bar{r}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Neither of these is a restriction of the technique.

Example 1: Homogenous Lossy Slab



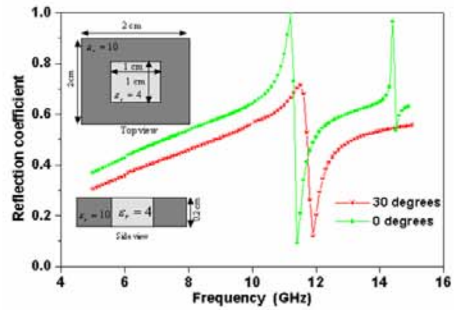
Reflection coefficients of the metal-backed lossy dielectric slab



Comparison of computing times

Numerical Results (2)

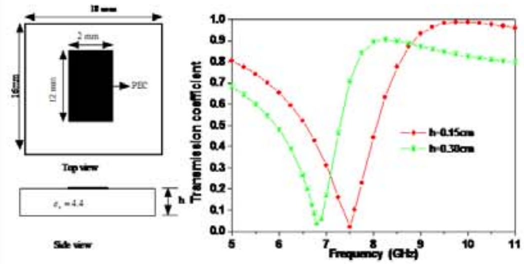
Example 2: Frequency-selective layer



T.F.Eibert, J.L.Volakis, D.R.Witon and D.V.Jackson, *IEEE Trans. Antennas Propaga.*(1999).

Numerical Results (3)

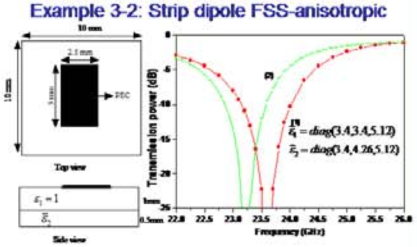
Example 3-1: Strip Dipole FSS-isotropic



A.L.P.S. Campos A.G.d'Assuncao and L.M.de.Mendonca. *IEEE Trans. On Microwave Theory Tech.* (2002)

Numerical Results (4)

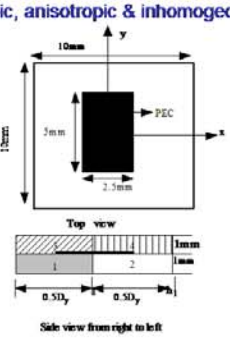
Example 3-2: Strip dipole FSS-anisotropic



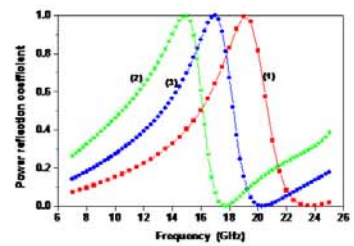
A.L.P.S. Campos A.G.d'Assuncao and L.M.de.Mendonca. *Microwave and Optical Technique letters*, vol. 33, pp. 57-61, April 5 2002. $\varepsilon_{xx} = \varepsilon_{yy}$

Numerical Results (5)

Example 3-3: Strip dipole FSS-isotropic, anisotropic & inhomogeous



Numerical Results (6)



Case 1: all regions are the same isotropic material (2,2,2)
Case 2: all regions are the same anisotropic material(3.4,3.4,5.12)
Case 3: the regions 1 and 3 are isotropic material (2,2,2)
the region 2 and 5 are anisotropic material (3.4,3.4,5.12)

Conclusions

- This paper discussed the FE-BI with brick elements accelerated by FFT and Ewald Transformation.
- The method was applied to 3D periodic structures composed of arbitrary materials.
- Numerical results confirm the accuracy, efficiency and flexibility of the method in dealing with structures with inhomogeneous and anisotropic materials.
- Current investigations are focusing on model order reduction.

Acknowledgements

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