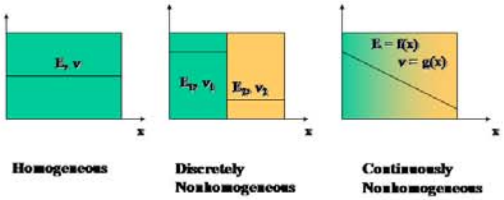
USE OF GRADED FINITE ELEMENTS TO MODEL WAVE PROPAGATION IN CONTINUOUSLY NONHOMOGENEOUS MATERIALS

P. Thamburaj (PD), M. H. Santare, and G. A. Gazonas (ARL)

University of Delaware • Center for Composite Materials • Department of Mechanical Engineering

Continuously Nonhomogeneous Materials – A Definition

What are continuously nonhomogeneous materials?



Homogeneous

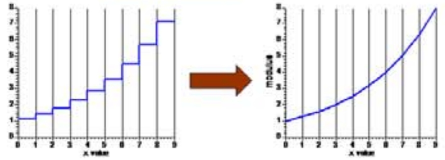
Discretely Nonhomogeneous

Continuously Nonhomogeneous

Motivation

Conventional finite elements
- Piece-wise property gradient
  - Mesh dependant
- Good approximation for global quantities
- Local quantities interpolated

- **Graded finite elements**
  - Continuous property gradient
  - Local evaluations
    - Failure analysis
    - Wave phenomena



Conventional finite elements

Graded finite elements

Current Aims

- Develop a graded finite element to model elastic stress wave propagation in materials with graded modulus and/or density
- Compare elastic stress wave results from graded element with conventional results
- Extend finite element formulation to include other continuously graded properties (inelastic behavior)

Elastic Wave Propagation – A 1D Example

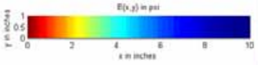
Decreasing E(x) across a 1D beam

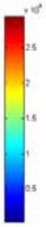
E0=300 E+6, r0=0.7333E-3

a=-0.9, m=2, n=0

8-node elements

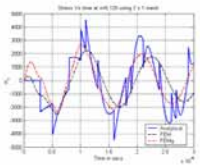
$$E(x) = E_0\left(a\frac{x}{l} + 1\right)^m$$

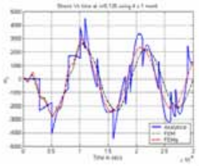




$$\rho(x) = \rho_0\left(a\frac{x}{l} + 1\right)^n$$

Stress vs Time - A 1D Example



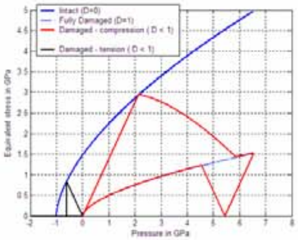


Dynamic Damage Model

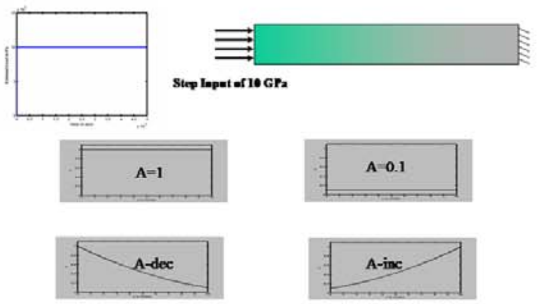
- Phenomenological model developed by G. R. Johnson and T. J. Holmquist (1994)

- Brittle materials
  - Gradual softening of material
  - Significant strength after fracture
  - Bulking effects
  - Strain rate effects

Johnson-Holmquist (JH2) Model

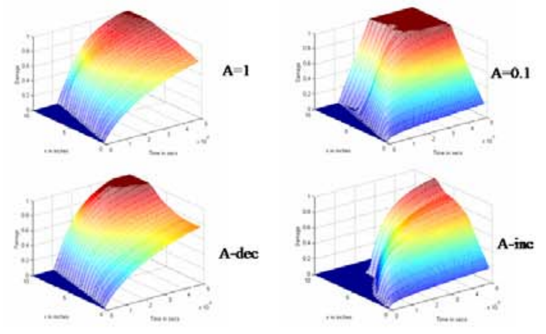


An Example – Input Step Load and Free- Fixed Edges



A – a parameter that affects the intact strength of the material
- The intact strength is directly proportional to A

An Example - Damage Vs time and x



The load is applied at x=0 and the edge x=L is fixed

Different Approaches to Damage Modeling

Phenomenological

Assumed constitutive response

Input macro experimental results

Curve-fit constitutive response to experiment

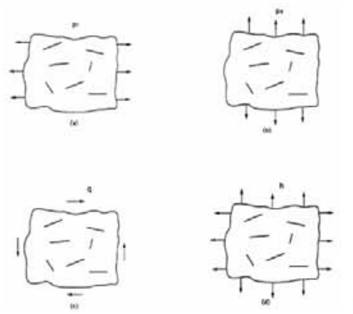
Micromechanical

Mechanical models of damage process

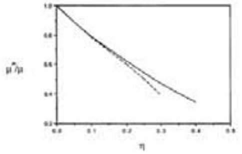
Input basic material properties

Predict macro response

Micromechanical Model

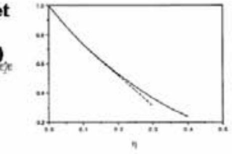


Micromechanical Model



$$\eta = \frac{Mc^2}{A}$$

- ISOTROPIC Santare et al (1995) v. Aboudi and Benveniste (1987)



Future Work

- Extend micro analysis to relate physical state of damage to dynamic material properties
- Adapt FE solutions to simulate micromechanical solutions.
- Develop numerical procedure to predict dynamic damage evolution (damage wave propagation)

Conclusions

- In elastic wave propagation, graded elements provide some small advantage over conventional elements
- This advantage is primarily wave speed resolution at large times using course meshes
- In inelastic wave propagation situations, the verdict is still out

Acknowledgements

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