WAVE DISPERSION IN CYLINDRICAL TUBES FOR SHPB TEST APPLICATION

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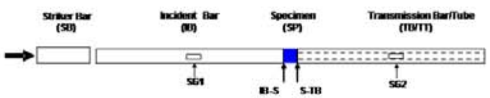
OBJECTIVE

To develop a wave dispersion correction algorithm for cylindrical tubes and apply this algorithm to the Split Hopkinson Pressure Bar (SHPB) test.

METHODOLOGY

Solve the M-H model and get the phase speeds, then do wave dispersion correction using such speeds. An finite element analysis (FEA) is next. If the predictions by FEA and M-H model are the same, the M-H model is proved to be valid.



TRADITIONAL DISPERSION ALGORITHM

To predict wave (strain vs. time) at the interfaces, Fourier Transform is done on the recorded strain data, then a dispersion correction on each harmonic component; finally do Inverse Fourier Transform and reconstruct the wave. This algorithm is valid only for cylindrical bars.

$$f(n\cdot\Delta T)=\frac{A_0}{2}+\sum_{k=1}^{N/2}[A_k\cos(k\omega_0 n\Delta T)+B_k\sin(k\omega_0 n\Delta T)]$$

$$A_0=\frac{2}{T}\int_0^T f(t)\,dt=\frac{2}{T}\sum_{n=1}^{N}f(n\Delta T)\cdot\Delta T$$

$$A_k=\frac{2}{T}\int_0^T f(t)\cos(k\omega_0 t)\,dt=\frac{2}{T}\sum_{n=1}^{N}f(n\Delta T)\cdot\cos(k\omega_0 n\Delta t)\Delta T$$

$$B_k=\frac{2}{T}\int_0^T f(t)\sin(k\omega_0 t)\,dt=\frac{2}{T}\sum_{n=1}^{N}f(n\Delta T)\cdot\sin(k\omega_0 n\Delta t)\Delta T$$

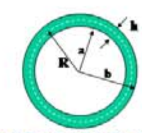
$$f'(n\cdot\Delta T)=\frac{A_0}{2}+\sum_{k=1}^{N/2}[A_k\cos(k\omega_0 n(\Delta T-\frac{\Delta z}{C_k}+\frac{\Delta z}{C_0}))+\sum_{k=1}^{N}[B_k\sin(k\omega_0 n(\Delta T-\frac{\Delta z}{C_k}+\frac{\Delta z}{C_0}))]$$

C_k is phase speed for harmonic wave component with frequency $k\omega_0$. C_k has been solved by Pochhammer and Cheer for cylindrical bars.

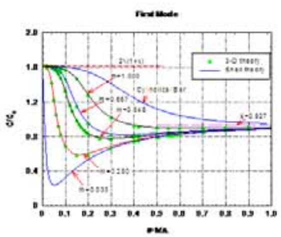
C_0 is fundamental speed.

MIRSKY-HERRMANN MODEL

Mirsky and Herrmann studied the wave propagation problem in infinitely long tubes. Frequency equations based on the 3-D model and shell model were derived. These two models gave very close results.



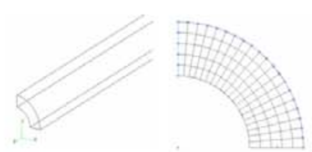
m=h/R, a=R-h/2, b=R+h/2



- Phase speed is a function of *m* and wavelength *Λ*.
- For small *m* values, speed first decreases and then increases, which is different from solid bars.
- When *δ* is 0, all the speed values are the same; when *δ* approached infinity, all the speed values approach another constant, which depends on the material's Poisson's Ratio *ν*.
- The M-H model gives the phase speed for harmonic wave propagation in infinitely long tubes.

FINITE ELEMENT ANALYSIS

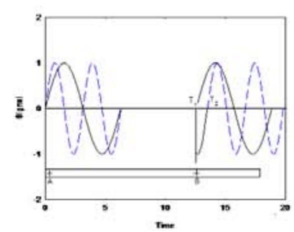
Finite element analysis (FEA) is performed and the results by FEA and the M-H prediction are compared to prove whether M-H model is usable for wave dispersion in SHPB.



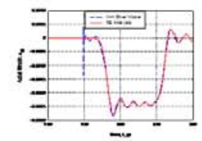
ABAQUS/Explicit is used to perform the analysis. Only one quarter of the tube is analyzed due to symmetry. A total of 60,000 8-node reduced integration brick elements are used.

NEW DISPERSION ALGORITHM

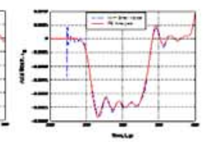
The traditional dispersion algorithm assumes continuous wave components in the bars. This assumption gives the wave dispersion correction, which is not accurate for tubes. A new algorithm considers the arriving time of each wave component separately.



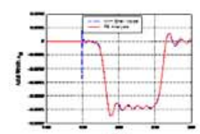
COMPARISONS



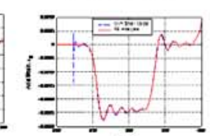
m=0.548 z=0.5L



m=0.548 z=0.75L



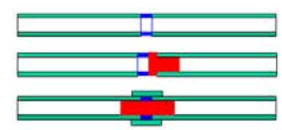
m=0.677 z=0.5L



m=0.677 z=0.75L

CONCLUSIONS

1. FEA and M-H model predictions are very close. Wave motion in tubes follow the first mode solution of M-H model.
2. In real test, tubes with lower *h* and higher *m* values should be used.
3. Various configurations for SHPB can be used:



ACKNOWLEDGEMENTS

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